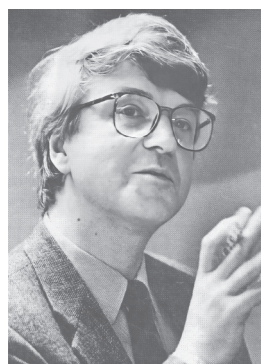# Sören Illman

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Professor Sören Illman passed away on 31 October 2023 in Helsinki, Finland. He was 80 years old. Having obtained his Master's degree at the University of Helsinki, he won a Fulbright Scholarship and moved to Princeton University, where he defended his doctoral thesis on equivariant algebraic topology in 1972. His advisor was Professor William Browder. Only three years later, he was appointed Professor of Mathematics at the University of Helsinki, where he remained until his retirement in 2011, staying on for a few more years as Professor Emeritus.

A large part of Illman's research pertained to the theory of transformation groups. Perhaps his most remarkable and best-known work is his contribution to Hilbert's fifth problem. The modern interpretation of this problem is hampered by the fact that the concept of a topological manifold was not exactly formalized at the time of Hilbert. Illman used the following interpretation.

Suppose G is a locally euclidean topological group and M is a locally euclidean space, and let

$$\Phi: G \times M \to M$$

be a continuous action of G on M. Is it then possible to choose the coordinates in G and M so that the action Φ becomes real analytic? According to an earlier interpretation, the same question was asked in the special case M = G. In this case, the answer is affirmative, as Gleason, Montgomery and Zippin proved in the 1950s. It is easily seen that the more general problem does not always have a solution, but Illman was able to prove the following:

Let G be a Lie group which acts on a C^1-smooth manifold M by a C^1-smooth and Cartan action. Then there exists a real analytic structure on M, compatible with the given smooth structure, such that the action of G becomes real analytic.

In short: if a Cartan Lie group action is differentiable, then it can be made real analytic. Here an action of G on M is said to be Cartan if each point x in M has a compact neighborhood A such that the set $G[A] = \{g \in G \mid gA \cap A \neq \varnothing\}$ is a compact subset of G.

Thanks to his solid mathematical research and the reputation he enjoyed in consequence of his publications, Illman received invitations to some of the most prestigious universities in the world. For periods up to an academic year at a time, he worked at Institute des Hautes Études Scientifiques, Bures-sur-Yvette, France; Math-

ematical Institute, University of Oxford, England; Institute for Mathematical Research, Eidgenössische Technische Hochschule, Zürich, Switzerland; Department of Mathematics, Yale University, New Haven; Research Institute for Mathematical Sciences, Kyoto University, Japan; Max Planck Institute for Mathematics, Bonn, Germany; Princeton University and the University of Cambridge, among others. He gave more than a hundred presentations at various conferences and meetings and was also one of the organisers of symposia on group actions on manifolds.

Sören Illman was elected a member of the Finnish Academy of Science and Letters in 1998. He was also a member of the Finnish Society of Sciences and Letters.

As an academic teacher, Illman contributed to the education of a group of algebraic topologists in Finland. Under his supervision, seven postgraduate students prepared and defended their doctoral theses, most of them concerning the theory of transformation groups. Even after this, he continued to support his former students, introducing them to the international community of transformation group researchers at various conferences.

As a colleague, Sören Illman was generous, considerate and supportive. His innate conviviality was evidently strengthened when he acclimatized to the academic environment in the United States, and after returning to Finland almost half a century ago, he provided a less formal alternative to the culture prevailing at the time in Finnish academia. Conversations with Sören were always interesting, because he had a deep insight into a variety of topics besides mathematics. During his long stays abroad, he studied the cultures of his host countries. This is how he started collecting ukiyo-e woodblock prints during his sojourn in Kyoto. He was also a cinephile and an ardent lover of opera music.

Erik Elfving and Kaj Malm