Delay differential Painlevé equations

Summary

An ordinary differential equation is said to have the Painlevé property if all of its solutions are single-valued about all movable singularities. This property has turned out to be a useful detector of integrable differential equations. With the discovery of integrable discrete systems, the need for formulating a discrete analogue of the Painlevé property became apparent. The idea was to try to find a discrete version of the Painlevé property which could be used to detect discrete integrable equations just as the (differential) Painlevé property can be applied for detecting integrable differential equations. Singularity confinement as proposed by Grammaticos, Ramani and Papageorgiou and the algebraic entropy of Hietarinta and Viallet were amongst the first candidates for this discrete Painlevé property. Halburd and the author applied a refined version of a method introduced by Ablowitz, Halburd and Herbst to single out difference equations of Painlevé type using the existence of finite-order meromorphic solutions as a criterion. We were able to single out all known difference equations of the Painlevé type from within a large class of difference equations, thus demonstrating that the existence of finite-order meromorphic solutions is a good candidate for the discrete Painlevé property. In a more recent work we have successfully applied a modified version of this approach to single out delay differential Painlevé equations from a natural class of delay differential equations. This method appears to be a good tool in the purpose of finding new delay differential Painlevé equations.