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ERRATUM TO "QUASICONFORMAL HARMONIC MAPPINGS WITH THE CONVEX HOLOMORPHIC PART"

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Corollary 2.7. *Given $R > 0$ let H be a conformal mapping in $\mathbf{D}$ such that $H(\mathbf{D})$ is a convex domain and $\mathbf{D}(H(0), R) \subset H(\mathbf{D})$. Then H is co-Lipschitz with*

$$\mathrm{L}^-(H) = \mathrm{D}^-(H) \geq \frac{R}{4}. \tag{2.19}$$

Proof. Under the assumption of the corollary we see that the mapping $\mathbf{D} \ni z \mapsto \tilde{H}(z) := H(z) - H(0)$ maps $\mathbf{D}$ onto a convex domain and $\mathbf{D}(0, R) \subset \tilde{H}(\mathbf{D})$. Since $\tilde{H}(0) = 0$ we conclude from [10, Corollary 3.1] (see also [5, Theorem 2.5]) that

$$|H'(z)| = |\tilde{H}'(z)| \geq \frac{R}{4}, \quad z \in \mathbf{D}.$$

Therefore $\mathrm{D}^-(H) \geq R/4$, and so the inequality in (2.19) holds. By Remark 2.6 we see that the equality in (2.19) holds. Therefore H is a co-Lipschitz mapping, which is our claim. □

Let us consider the following deformations of a harmonic mapping $F = H + \overline{G}$ in $\mathbf{D}$,

$$\mathbf{D} \ni z \mapsto F_\varepsilon(z) := H(z) + \varepsilon\overline{G(z)}, \quad \varepsilon \in \mathbf{C}. \tag{2.20}$$

Using now the decomposition (2.2) we derive the following theorem.

Theorem 2.8. *Let $F = H + \overline{G}$ be a sense-preserving harmonic mapping in $\mathbf{D}$. Suppose that H is injective, $H(\mathbf{D})$ is a rectifiably M-arcwise connected domain with a given $M \geq 1$ and that F is not a conformal mapping. Then for every $\varepsilon \in \mathbf{D}(1/M\|\mu_F\|_\infty)$, F_ε is a quasiconformal harmonic mapping. Moreover, if $M = 1$, then F_ε is co-Lipschitz.*

Proof. Fix $\varepsilon \in \mathbf{D}(1/M\|\mu_F\|_\infty)$. By setting $H(\mathbf{D}) \ni z \mapsto \phi(z) := \overline{\varepsilon} G \circ H^{-1}(z)$, we see that for every $z \in H(\mathbf{D})$,

$$|\phi'(z)| = \left|\overline{\varepsilon}\frac{G'(H^{-1}(z))}{H'(H^{-1}(z))}\right| = |\varepsilon||\mu_F(H^{-1}(z))| \leq |\varepsilon|\|\mu_F\|_\infty. \tag{2.21}$$

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Hence $M\,\mathrm{D}^+(\phi) \leq M|\varepsilon|\|\mu_F\|_\infty < 1$. From Lemma 2.4 it follows that $\mathrm{I}[\phi]$ is bi-Lipschitz, and so $\mathrm{I}[\phi]$ is quasiconformal. Since $F_\varepsilon = \mathrm{I}[\phi] \circ H$, F_ε is a quasiconformal mapping as a composition of quasiconformal ones. Suppose now that $M = 1$, i.e., $H(\mathbf{D})$ is a convex domain. By the conformality of H, $\mathbf{D}(H(0), R) \subset H(\mathbf{D})$ for a certain positive number R. Then by Corollary 2.7 we see that H is a co-Lipschitz mapping. Therefore F_ε is a co-Lipschitz mapping as a composition of co-Lipschitz ones, which proves the theorem. □

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