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ADDENDUM TO "NEW CHARACTERIZATIONS OF BERGMAN SPACES"

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Two problems have been brought to our attention since the publication of our paper "New characterizations of Bergman spaces" [5] which will be referred to as "the paper" in what follows. The first issue concerns a result which we thought was well-known but was actually not quite so. The second issue concerns a lack of details in a major step of the proof of Theorem 2 in the paper. We will clarify these issues in this addendum.

In addition, Kwon sent [3] to the first named author before our paper was accepted for publication, but we failed to acknowledge Kwon's paper. We wish to apologize here. Kwon's paper [3] proves the one-dimensional case of our main results for Bergman spaces with more general weights under the additional assumption that $f(0) = 0$. The one dimensional case of the Littlewood–Paley inequality can also be found in [1]. Related work for Hardy spaces on the unit ball can be found in [6].

We asserted in the paper that Lemma 9 could be found in [4]. This is not the case. The first inequality in Lemma 9,

$$\int_{\mathbf{B}_n} |f|^p\,dv \le C\left[|f(0)|^p + \int_{\mathbf{B}_n} |f(z)|^{p-2}|\widetilde{\nabla} f(z)|^2\,dv(z)\right], \tag{1}$$

was not used anywhere in the paper. However, the second inequality in Lemma 9,

$$|f(0)|^p + \int_{\mathbf{B}_n} |f(z)|^{p-2}|\widetilde{\nabla} f(z)|^2\,dv(z) \le C\int_{\mathbf{B}_n} |f|^p\,dv, \tag{2}$$

was used in the paper to prove the inequality

$$|f(0)|^p + I_4(f) \le C I_1(f).$$

To prove (2), we use the identity

$$\int_{\mathbf{B}_n} |f|^p\,dv = |f(0)|^p + c_{p,n}\int_{\mathbf{B}_n} |\widetilde{\nabla} f(z)|^2 |f(z)|^{p-2} G_1(z)(1-|z|^2)^{-n-1}\,dv(z), \tag{3}$$

where

$$G_1(z) = \int_{|z|}^1 \frac{(1-t^2)^{n-1}(1-t^{2n})}{t^{2n-1}}\,dt.$$

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Identity (3), which was stated as Exercise 4.5 in [7], follows from Theorem 4.23 of [7] and integration in polar coordinates. Since

$$G_1(z) \geq \int_{|z|}^{1} \frac{(1-t)^{n-1} 2nt^{2n-1}(1-t)\,dt}{t^{2n-1}} = \frac{2n}{n+1}(1-|z|)^{n+1},$$

we obtain inequality (2).

Taking $q = 2$ in Theorem 2 of the paper, we obtain Lemma 9 as a consequence.

There is a second point in the paper that warrants clarification. Namely, we proved that for $p < q < p+2$,

$$cI_1(f) \leq |f(0)|^p + I_2(f)^{\frac{1}{r}} I_1(f)^{\frac{1}{s}},$$

and we then wrote that from this one easily deduces

$$I_1(f) \leq C[|f(0)|^p + I_2(f)].$$

This is true if f is holomorphic in a neighborhood of the closed ball. If f is arbitrary, we want to apply this fact to the functions

$$f_\rho(z) = f(\rho z), \quad 0 < \rho < 1,$$

to get

$$\begin{aligned} &\frac{1}{\rho^{2n+2\alpha}} \int_{\rho \mathbf{B}_n} |f(w)|^p (\rho^2 - |w|^2)^\alpha \, dv(w) \\ &\leq C|f(0)|^p + \frac{C}{\rho^{2n+2\alpha+2q}} \int_{\rho \mathbf{B}_n} |Rf(w)|^q |f(w)|^{p-q} (\rho^2 - |w|^2)^{q+\alpha} \, dv(w). \end{aligned}$$

When $q + \alpha \geq 0$, we let $\rho \to 1^-$ and apply Fatou's lemma on the left hand side and the monotone convergence theorem on the right to get the desired result. However, if $q + \alpha < 0$, which implies $p < 1$, we cannot apply the monotone convergence theorem. In this case, we can use a trick due to Kwon [2, 3] as follows. From the Littlewood–Paley type inequality

$$M_p^p(r, f) \leq C|f(0)|^p + C \int_{\rho \mathbf{B}_n} (\rho - |z|)^{p-1} |Rf(z)|^p \, dv(z), \quad p < 1, \ \tfrac{1}{2} < \rho < 1,$$

we get as in [2, 3]

$$\int_{\rho \mathbf{B}_n} |f(z)|^p (1 - |z|^2)^\alpha \, dv(z) \leq C|f(0)|^p + C \int_{\rho \mathbf{B}_n} |Rf(z)|^p (1 - |z|^2)^{p+\alpha} \, dv(z).$$

If we replace $I_1(f)$ and $I_2(f)$ by $I_1(\rho, f)$ and $I_2(\rho, f)$, respectively, where

$$I_1(\rho, f) = \int_{\rho \mathbf{B}_n} |f(z)|^p \, dv_\alpha(z),$$

and

$$I_2(\rho, f) = \int_{\rho \mathbf{B}_n} |f(z)|^{p-q} |(1 - |z|^2) Rf(z)|^q \, dv_\alpha(z),$$

we obtain

$$I_1(\rho, f) \leq C \left[|f(0)|^p + I_2(\rho, f) \right].$$

Let $\rho \to 1$ then we obtain the desired result.

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